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ABSTRACT

Recent results on digital straightness and convexity are reviewed, and it is shown that the criteria for a set of lattice points to be the digitization of a convex set, or for a digital arc to be the digitization of a straight line segment, depend critically on the definition of digitization that is used.

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HOW A DIGITAL COMPUTER CAN TELL THAT A STRAIGHT LINE IS STRAIGHT

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1. Introduction. Image processing and pattern recognition [1] are often concerned with classifying shapes or patterns that appear in pictures, and the classification is often based on geometrical properties of the patterns. For example, in a picture of a nuclear bubble chamber, we may want to classify the particle tracks as being straight line segments, circular arcs, etc., in order to identify the particles that gave rise to these tracks. As another example, in a photomicrograph of a blood smear, we may want to determine whether the nucleus of a white blood cell is convex or has concavities in order to identify which type of cell it is.

What makes such tasks nontrivial is that computers can only deal with pictures that have been "digitized," i.e., converted into arrays of lattice points, and it is not always obvious how to recognize that a set of lattice points must have arisen from a real pattern that has a given geometric property. For example, how do we characterize sets of lattice points that are the digitizations of real straight line segments? This and some related questions will be discussed in this paper. In order to treat them, we must first define more precisely what we mean by "digitization," and introduce some basic "digital picture" terminology. As we shall see, the results depend strongly on the definitions of digitization that we use.

2. Digitization of bounded subsets

Let S be a bounded subset of the plane. For purposes of computer analysis, it is customary to represent S by a finite set of lattice points, i.e., points with integer coordinates. This set $\hat{S}$ is called the digital image of S, and the mapping that takes S into $\hat{S}$ is called digitization.

$\hat{S}$ can be defined in a number of ways; we list several of them here:

a) $\hat{S}$ is the set of lattice points contained in S; this is called the subset digitization of S.

b) $\hat{S}$ is the set of lattice points such that S comes closer than city block distance $\frac{1}{2}$ to them - i.e., $\{(i,j)\,|\,\exists(x,y)\in S: \max(|x-i|,|y-j|)<\frac{1}{2}\}$. This is called the open cell digitization of S. (If we imagine an open unit square ["cell"] P° centered at each lattice point P, we have $P\in\hat{S}$ iff $S\cap \overset{\circ}{P}\neq\emptyset$.)

b') Analogous to (b), using half-open cells P^*, e.g., $i-\frac{1}{2}\le x<i+\frac{1}{2}$, $j-\frac{1}{2}\le y<j+\frac{1}{2}$.

b") Analogous to (b), using closed cells $\overline{P}$.

Note that by definitions (a-b), a nonempty set S can have an empty digitization. In the course of this paper we will discover other advantages and disadvantages of the various definitions.

A set T of lattice points is called 8-connected if for all P,Q in T there exists a finite sequence $P=P_0,P_1,\ldots,P_n=Q$ of points of T such that P_i is a horizontal, vertical, or diagonal neighbor (for brevity: an 8-neighbor) of P_{i-1}, $1\le i\le n$.

If only horizontal and vertical neighbors ("4-neighbors") are allowed, we call T <u>4-connected</u>.

<u>Proposition 1</u>. If S is arcwise connected, then by definition (a) or (b), $\hat{S}$ need not be 8-connected; by definition (b'), it must be 8-connected; and by definition (b"), it must be 4-connected. ||

Further properties of 4- and 8-connectedness are treated in [2,3].

3. Digitization of arcs

None of the definitions of digitzation given in Section 2 is entirely satisfactory if S is an arc. As we traverse an arc A, we would like to define a sequence of lattice points belonging to the digitization of A, and we would also like the digitization of an arc to be connected. The connectedness requirement immediately rules out the subset and open cell definitions (a-b); while if we use the closed cell definition (b"), the lattice points of $\hat{A}$ do not occur in a simple sequence; when A leaves a cell through one of its corners, three new lattice points (at the centers of the other cells sharing that corner) appear simultaneously on $\hat{A}$. This leaves only the half-open cell definition (b'), for which the lattice points of $\hat{A}$ do in fact occur in sequence as A is traversed. Each of these points is an 8-neighbor of the preceding one, so that $\hat{A}$ is determined by specifying a starting point and a sequence of moves from neighbor to neighbor [4].

This approach provides a compact way of specifying $\hat{A}$, but it is somewhat wasteful in the sense that diagonal moves occur with zero probability; when an arc leaves a cell, it almost certainly does so along a side, not at a corner. For this reason, a different definition of digitization has historically been used for arcs, which we may call <u>grid digitization</u>. Imagine the lattice points joined by a grid of lines; thus as we traverse A, we cross a succession of grid lines. (Note that $\hat{A}$ can be empty if A never crosses a grid line; but then A always stays inside a single cell.) Whenever we cross a grid line, the lattice point (=grid line intersection) closest to the crossing point becomes a point of $\hat{A}$. If we cross halfway between two lattice points, we resolve the tie by using, e.g., the lattice point that lies to the right of A (in the sense that we are traversing it)*. This grid digitization evidently defines a sequence of lattice points in $\hat{A}$ as A is traversed, each an 8-neighbor of the preceding; but it is easily seen that diagonal neighbors now have nonzero probability. A further advantage of grid digitization over cell digitization will become apparent in the next section.

*Alternatively, we could resolve ties by rounding, but as we shall see, the method defined here is preferable.

4. Digital arcs

The finite set of lattice points B is called a digital arc if

a) B is connected

b) All but two points of B have exactly two neighbors in B

c) Two points of B, called the endpoints, have exactly one neighbor in B

Note that this is two definitions in one, depending on whether we use the 4- or 8-definition for "neighbor" and "connected."

Proposition 2. If B is a digital arc, and we use the subset, open cell, or grid definition of digitization, then there exists an arc A such that $B=\hat{A}$.

Proof: If we start from one of the endpoints, go to its neighbor, then go to the other neighbor of that neighbor (if any), and repeat the process, we can keep on until we reach a point that has no other neighbor, which must be the other endpoint. It is not hard to see that since B is connected, the sequence of points defined in this way is all of B. (If a point in the sequence were connected to a point not in the sequence, some point in the sequence would have to have a third neighbor.) The polygonal arc A joining this succession of points then evidently has B as its digitization by the three definitions mentioned. Note that the Proposition is not true for the other two definitions - e.g., the digital 8-arc $\{(0,0), (1,1)\}$ is not the closed cell digitization of any arc. ||

Unfortunately, if A is an arc, $\hat{A}$ need not be a digital arc, since $\hat{A}$ may touch itself if A passes sufficiently close to itself. However, we can prove

<u>Proposition 3</u>. If A is a straight line segment, and we use the grid definition of digitization, then $\hat{A}$ is a digital 8-arc.

<u>Proof</u>: As we move along A, we visit the points of $\hat{A}$ in succession. It is not hard to see that the successive points of $\hat{A}$ (if distinct) are 8-neighbors, and that two points of $\hat{A}$ cannot be 8-neighbors unless they are successive. ||

Proposition 3 does not hold if we use the subset or cell digitizations, or even if we use the grid method but resolve ties by rounding. For the subset or open cell method, $\hat{A}$ can evidently be empty; and for the closed cell method, the line $x=i\pm\frac{1}{2}$ or $y=j\pm\frac{1}{2}$ has a double-thickness digitization. Even for the half-open cell method, let A be the line through $(\frac{1}{2},\frac{1}{2})$ with slope $-45°$; then $\hat{A}$ defined by the half-open cell method has a digitization that is a 4-arc, not an 8-arc, since it contains the lattice points $\ldots,(1,0),(1,1),(0,1),(0,2),(-1,2),\ldots$. Similarly, let A be the line through $(\frac{1}{2},0)$ with slope 45°, and let $\hat{A}$ be defined by the grid method but with ties resolved by rounding down; then $\hat{A}$ is a 4-arc but not an 8-arc, since it contains the lattice points (0,0),(1,0),(1,1),(2,1),... (The same example works if we round up rather than down; and if we round up in one coordinate and down in the other, we can give an analogous example using a line of slope -45°.)

We are now ready to consider the question posed in the title of this paper: Given a digital arc, how can we tell whether it is the digitization of a straight line segment? Note that any digital arc is always the digitization of things that are not straight line segments, but we want to know when it is also the digitization of a straight line segment.

5. Straight digital arcs

A digital 8-arc B will be called <u>straight</u> if there exists a straight line segment A such that $\hat{A}=B$ (using grid digitization).

<u>Theorem 4</u>. The following properties of the digital arc B are equivalent:

a) B is straight

b) There exists no triple of collinear lattice points P,Q,R (with Q between P and R) such that P,R are in B but Q is not

c) For any lattice points P,R of B, and any point (x,y) on the line segment $\overline{PR}$, there exists a lattice point (i,j) of B such that $\max(|x-i|,|y-j|)<1$. ||

It is not hard to show that if B is straight, it has properties (b-c). Conversely, we can easily show that if B has property (b) or (c), its sequence of moves from neighbor to neighbor can involve at most two directions, which can only differ by 45°, and that at least one of these directions has only isolated occurrences in the sequence; thus the sequence consists of runs in a given direction, separated by single moves in an adjacent direction. Now if property (b) or (c) holds for B, it also holds for the digital arc B' obtained by deleting the last point of B; hence by induction, B' is straight, say $B'=\hat{A}'$; and whether the last point P of B extends a run or starts a new run, one can find an A' such that P is on the digitization of an extension of A'. For the details of a proof that (a) and (c) are equivalent, see [5]. Other aspects of digital straightness are treated in [6-12].

A set of lattice points for which (b) holds will be said to have the <u>collinearity property</u>, and a set for which (c) holds will be said to have the <u>chord property</u>. At first glance, (b) and (c) seem tedious to verify; but in fact, they need only be checked for pairs P,R of lattice points of B that are run ends, and (as regards (c)) for points (x,y) that have the same coordinate as a run end; the details are straightforward. Theorem 4 is not true for other definitions of digitization; $\{(0,0),(1,1)\}$ and $\{(0,1),(1,0)\}$ are digital 8-arcs, and evidently have properties (b-c), but as we have already seen, by the other definitions they are not both digitizations of straight line segments.

6. Digital convexity

The conditions of Theorem 4 turn out to be of interest for other reasons; in fact, they are precisely the conditions for a set of lattice points to be the digitization of a convex set, if we use the subset definition of digitization. To begin with, it can be shown [13-17] that

Theorem 5. The following properties of a finite set T of lattice points are equivalent:

a) T has the collinearity property

b) T has the chord property

c) The convex hull of T contains no lattice point in the complement of T. ||

T is called *digitally convex* if there exists a convex set S such that $\hat{S}=T$.

Theorem 6. T is digitally convex (using the subset definition of digitization) iff it has the properties of Theorem 5. ||

For other definitions of digitization, Theorem 6 does not hold. If we use the cell definitions, the conditions of Theorem 5 are necessary but not sufficient. As an example, let T be

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Then T has the properties of Theorem 5, but is not digitally convex by any of the cell definitions. Partial characterizations of digital convexity using various definitions of digitization can be found in [18-25].

We can also prove

Theorem 7. T is digitally convex (subset definition) iff for any two lattice points P,Q of T there exists a straight digital 8-arc B such that $P,Q \in B \subseteq T$. ||

The convex hull property in Theorem 5 can be used as the basis of an algorithm for determining whether a given set T of lattice points is digitally convex We first construct the convex hull of T; in fact, it suffices to construct the convex hull of the set of "corner points" of T (points of T that have two horizontal or vertical neighbors in the complement of T that are diagonally adjacent to each other). We then check whether the convex hull contains a point of the complement; in fact, it suffices to check whether it contains a "corner point" of the complement. If we represent T by a scheme called run-length coding (see [1]), the entire process can be carried out in time on the order of M, the image side length (i.e., T is contained in an M by M array of lattice points). A similar procedure can be used to determine whether T is a straight digital arc: first verify that it is a digital arc, then check that it is convex.

It should be noted that the situation is more complex in three dimensions [26]. For example, it can be shown that when we use a method analogous to open cell digitization, the chord property is sufficient but not necessary for a set of lattice points in three dimensions to be the digitization of a convex object. Three-dimensional digital geometry is a subject of rapidly growing interest with the increasing need to process three-dimensional data arrays, e.g., as obtained by computed tomography.

7. Concluding remarks

Determining whether a sequence of lattice points could be the digitization of a straight line segment, or a set of lattice points the digitization of a convex object, is of practical interest in digital image processing and pattern recognition. These problems turn out to have neat solutions for some definitions of digitization, but not for others. Thus the method of digitization used to represent planar subsets in a computer can have unexpected implications with respect to determining geometric properties of the subsets.

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